A Survey on the Structured Singular Value

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Abstract

The structured singular value, μ, is an important linear algebra tool to study a class of matrix perturbation problems. It is useful for analyzing the robustness of stability and performance of uncertain, (nominally) linear systems. Computation of (M) is difficult, and usually, upper and lower bounds are all that can be reliably computed. Upper bounds give conservative estimates of the sizes of allowable perturbations. The maximum singular value of a matrix M is an upper bound for (M). As an upper bound, it can be improved by finding a transformations to the data (ie. M) which do not change the structured singular value, but do reduce the maximum singular value. Typically, upper bound algorithms involve searches over sets of transformations to yield the tightest bound. Lower bound algorithms produce small, destabilizing perturbations. In general, the algorithms are intelligent searches for minimum-norm solutions to multivariable polynomial equations, and are based on various optimality conditions that hold at the global (and, unfortunately, some local) minima. This paper reviews the current methods to compute both of these types of bounds, covering theoretical justification and extensive numerical experience with the various algorithms.